Technical Note - TN12

Electric and magnetic field sensor and integrator equations

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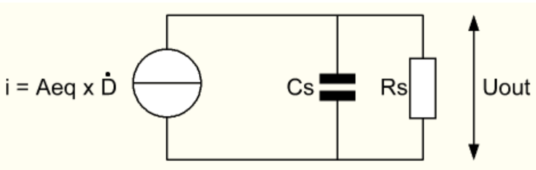
1. Equations of the derivative electric field sensor

The charges induced on the sensor are flowing through the impedance of the measuring system. The current produced by the sensor is proportional to the equivalent surface A_{eq} and to the first time derivative of the charges displacement.

The equivalent circuit of the sensor is the following:



and the equation is:

$$i(t) = C_s \frac{\partial v(t)}{\partial t} + \frac{v(t)}{R_s} = A_{eq} \frac{\partial D(t)}{\partial t}$$

where:
R_s is the impedance seen by the sensor
C_s is the capacitance of the sensor
A_{eq} the equivalent surface
D the electric displacement

Using the Laplace transform, the equation becomes:

$$A_{eq}\, sD(s) = C_s s\, v(s) + \frac{v(s)}{R_s}$$

where:

$$s = j\omega \quad , \quad |s| = |j\omega| = \omega \quad , \quad \omega = 2\pi f$$

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So the response of the sensor is given by:

$$v_{out}(s) = \frac{R_s\ A_{eq}\ sD(s)}{sR_sC_s + 1}$$

1) For the differentiating mode (low frequency domain: $\omega R_s C_s << 1$):

$$v_{out}(s) = R_s\ A_{eq}\ sD(s)$$

with $s = j\omega$, $|s| = |j\omega| = \omega$, $\omega = 2\pi f$ and $D = \varepsilon_0 E$:

$$|v_{out}(f)| = R_s\ A_{eq}\ 2\pi f\ \varepsilon_0 |E(f)|$$

Remark: $F_c = \dfrac{1}{2\pi R_s C_s}$

or in the time domain:

$$v_{out}(t) = R_s\ A_{eq} \frac{\partial D(t)}{\partial t}$$

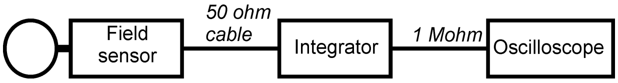
2) For the self integrating mode (high frequency domain: $\omega R_s C_s >> 1$):

$$v_{out}(s) = \frac{A_{eq}\ D(s)}{C_s}$$

or in the time domain:

$$v_{out}(t) = \frac{A_{eq}\ D(t)}{C_s}$$

2. Integration of the signal



The transfer function of the integrator is

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{1}{sR_iC_i + 1}$$

where:
R_iC_i is the time constant of the integrator.

So the response of the full system (sensor + integrator) is:

$$v_{out}(s) = \frac{R_s\ A_{eq}\ sD(s)}{sR_sC_s + 1} \frac{1}{sR_iC_i + 1}$$

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With both following conditions:

- differentiating mode of the sensor (low frequency domain: $\omega R_s C_s << 1$);
- frequency of the signal to be integrated large enough ($\omega R_i C_i >> 1$);

the response of the full system becomes:

$$v_{out}(s) = \frac{R_s \, A_{eq} \, D(s)}{R_i C_i}$$

or in the time domain:

$$v_{out}(t) = \frac{R_s \, A_{eq} \, D(t)}{R_i C_i}$$

or in relation with the electric field (V/m):

$$v_{out}(t) = \frac{R_s \, A_{eq} \, \varepsilon_0 \, E(t)}{R_i C_i}$$

with $D = \varepsilon_0 E$ in free space

3. Example of calculation

The following calculation is carried out for a SGE2G sensor connected through an ITR1-2U integrator to the 1 MΩ input of an oscilloscope.

The datas are the following:
R_s is the impedance seen by the sensor (50 Ω)
A_{eq} the equivalent surface (5.4 x 10^{-3} m^2)
$R_i C_i$ time constant of the integrator (1.2 µs)
ε_0 = 8.854 x 10^{-12} $AsV^{-1}m^{-1}$

$$v_{out}(t) = \frac{50 \times 5.4 \times 10^{-3} \times 8.854 \times 10^{-12}}{1.2 \times 10^{-6}} \times E(t)$$

This gives a correction factor of the sensor of 2 mV / kV/m.

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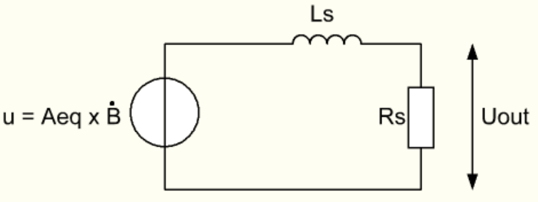
4. Equations of the derivative magnetic field sensor

The magnetic induction generates a voltage induced proportional to the equivalent surface A_{eq} and to the first time derivative of the magnetic induction.

The equivalent circuit of the sensor is the following:



and the equation is:

$$v(t) = L_s \frac{\partial i(t)}{\partial t} + R_s i(t) = A_{eq} \frac{\partial B(t)}{\partial t}$$

where:
R_s is the impedance seen by the sensor
L_s is the inductance of the sensor
A_{eq} the equivalent surface
B the magnetic induction

Using the Laplace transform, the equation becomes:

$$A_{eq}\, sB(s) = L_s s\, i(s) + R_s i(s)$$

Then the response of the sensor is given by :

$$v_{out}(s) = \frac{A_{eq}\, sB(s)}{\frac{sL_s}{R_s} + 1}$$

1) For the differentiating mode (low frequency domain: $\omega L_{s/} R_s$ << 1):

$$v_{out}(s) = A_{eq}\, sB(s)$$

with $s = j\omega$, $|s| = |j\omega| = \omega$, $\omega = 2\pi f$ and $B = \mu_0 H$:

$$|v_{out}(f)| = A_{eq}\, 2\pi f\, \mu_0 |H(f)|$$

Remark: $F_c = \dfrac{R_s}{2\pi L_s}$

or in the time domain:

$$v_{out}(t) = A_{eq} \frac{\partial B(t)}{\partial t}$$

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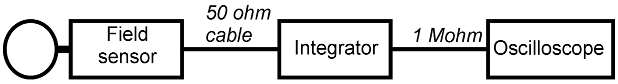
2) For the self integrating mode (high frequency domain: $\omega L_s/R_s >> 1$):

$$v_{out}(s) = \frac{A_{eq}\ B(s)\ R_s}{L_s}$$

or in the time domain:

$$v_{out}(t) = \frac{A_{eq}\ B(t)\ R_s}{L_s}$$

5. Integration of the signal



The transfer function of the integrator is

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{1}{sR_iC_i + 1}$$

where:
R_iC_i is the time constant of the integrator

So the response of the full system (sensor + integrator) is:

$$v_{out}(s) = \frac{A_{eq}\ sB(s)}{\frac{sL_s}{R_s} + 1}\ \frac{1}{sR_iC_i + 1}$$

With both following conditions:

- differentiating mode of the sensor (low frequency domain: $\omega L_s/R_s << 1$);
- frequency of the signal to be integrated large enough ($\omega R_iC_i >> 1$);

the response of the full system becomes:

$$v_{out}(s) = \frac{A_{eq}\ B(s)}{R_iC_i}$$

or in the time domain:

$$v_{out}(t) = \frac{A_{eq}\ B(t)}{R_iC_i}$$

or in relation with the magnetic field (in A/m):

$$v_{out}(t) = \frac{A_{eq}\ \mu_0\ H(t)}{R_iC_i}$$

with $H = \mu_0\ B$ in free space

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6. Example of calculation

The following calculation is carried out for a SGM1-8G sensor connected through an ITR1-2U integrator to the 1 MΩ input of an oscilloscope.

The datas are the following:

R_s is the impedance seen by the sensor (50 Ω)

A_{eq} the equivalent surface (1 x 10^{-3} m^2)

R_iC_i time constant of the integrator (1.2 μs)

$\mu_0 = 1.256 \times 10^{-6}$ $AsV^{-1}m^{-1}$

$$v_{out}(t) = \frac{1 \times 10^{-3} \times 1.256 \times 10^{-6}}{1.2 \times 10^{-6}} \times H(t)$$

This gives a correction factor of the sensor of 1 mV / A/m.

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7. Appendix 1 : graphical results

The following graphics show examples of calculations applied on a typical electric derivative sensor.

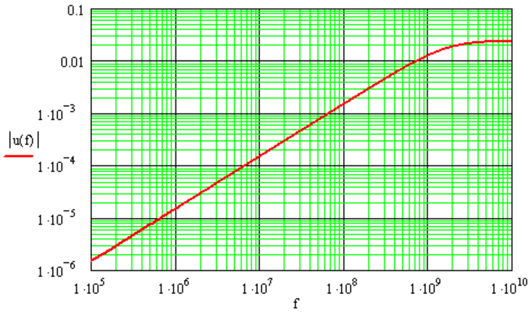

Figure 1 : Response of the sensor alone (relation output voltage / E field)

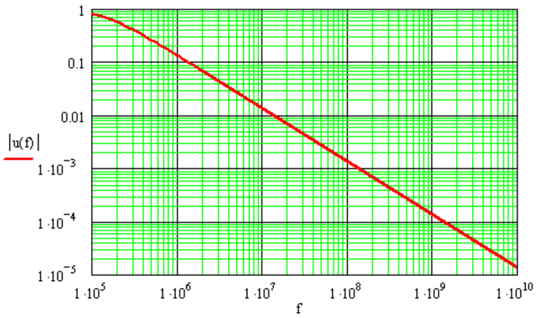

Figure 2 : Response of the integrator alone (output voltage / input voltage)

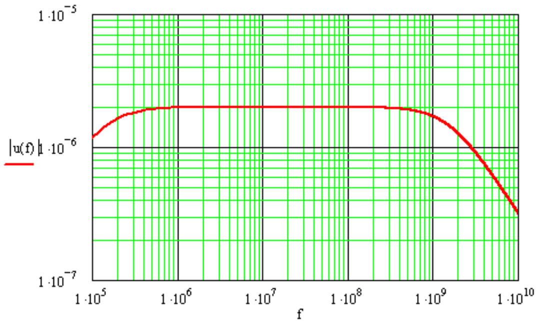

Figure 3 : Response of the full system (relation output voltage / E field)

The figure 1 shows both frequency domains: 1) the differentiating low frequency region and 2) the high frequency self integrating region (above about 3 GHz in this example). The sensor must be designed to have a transition frequency between these 2 regions high enough.

The combination of the sensor with the integrator gives an intermediate frequency range for which the response of the full system is flat. Therefore the integrator must be specially designed for the sensor. The time constant has also a direct influence on the level of the flat region, that is to say on the sensitivity of the whole system.

Remark: the resonance frequencies of the sensor and of the integrator are not represented here.

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